05 – Divergence and Command Reversal

Vibraciones y Aeroelasticidad
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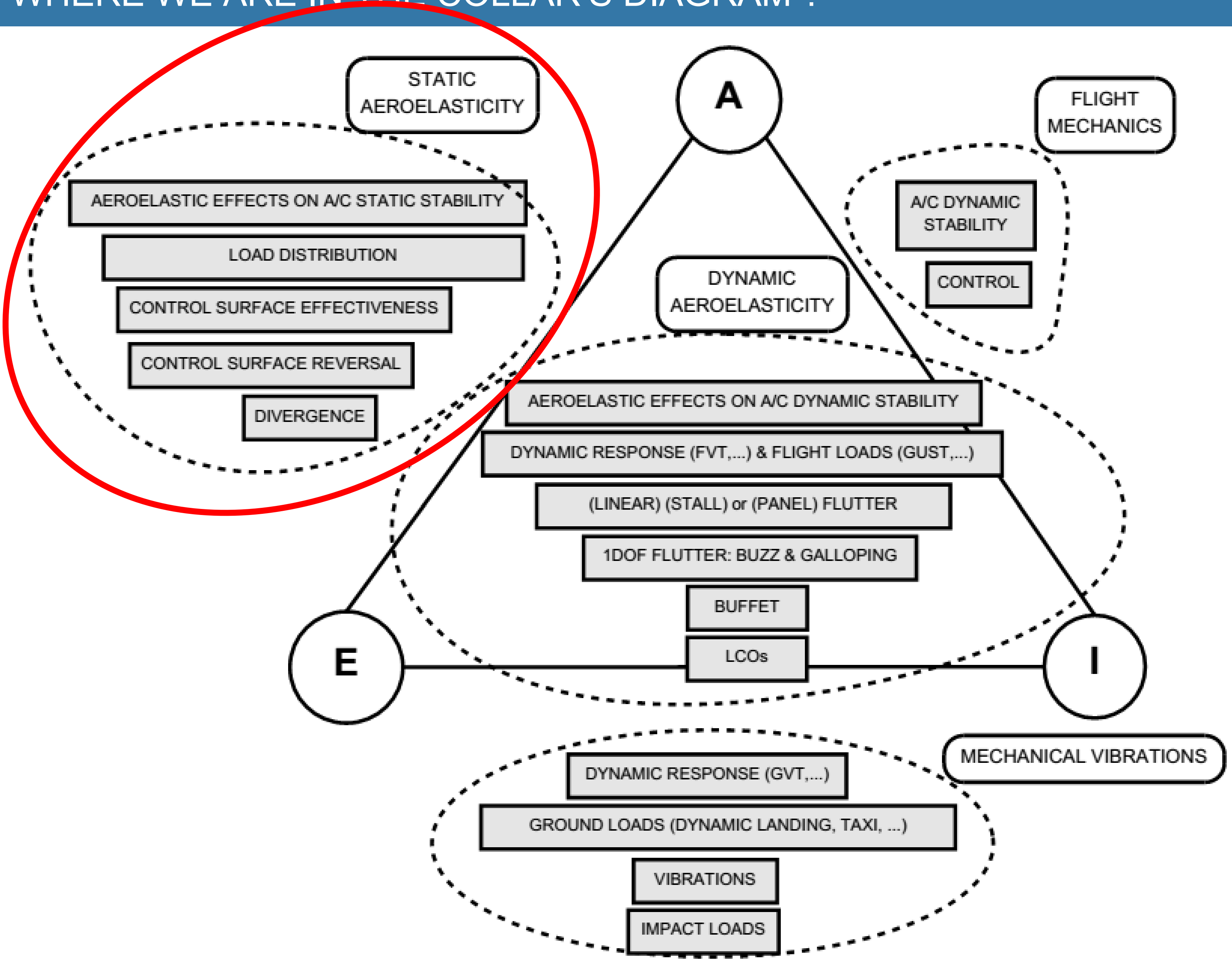
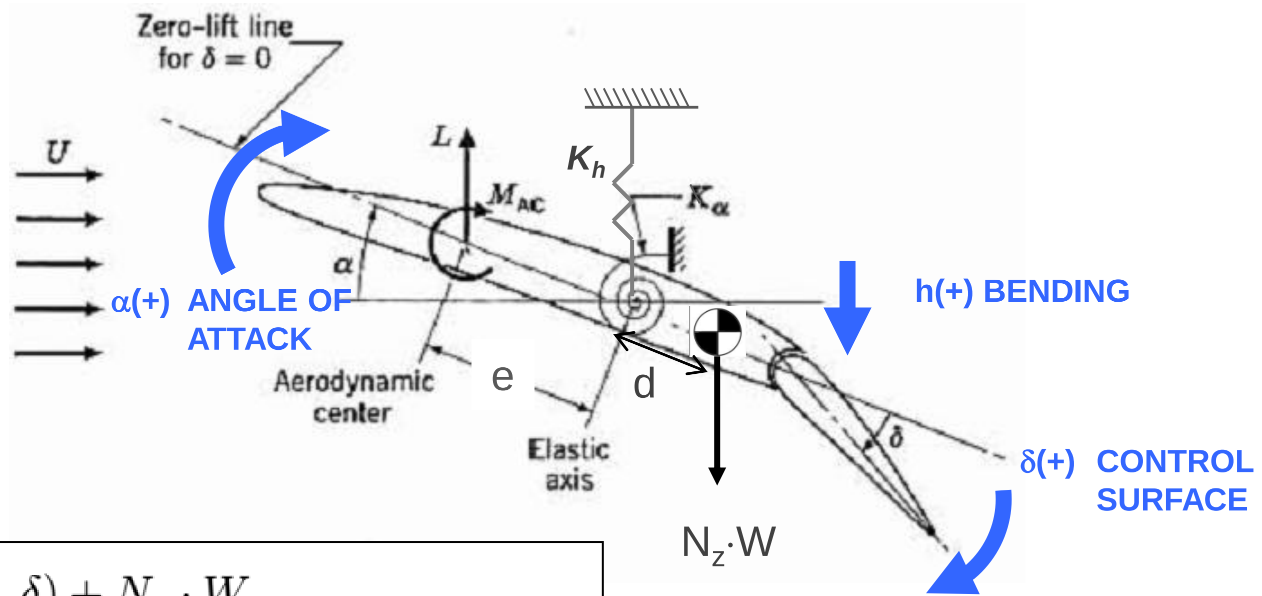
STATIC AEROELASTICITY
WHERE WE ARE IN THE COLLAR'S DIAGRAM ?



- Static aeroelastic equations of the "Typical Section" including a Control Surface (CS) deflection:



$$
\begin{aligned}
K_h \cdot h &= -L(\alpha,\delta) + N_z \cdot W \\
K_\alpha \cdot \Delta\alpha &= M_{AC}(\alpha,\delta) + e \cdot L(\alpha,\delta) + d \cdot N_z \cdot W
\end{aligned}
$$

$$\alpha = \alpha_r + \Delta\alpha_h + \Delta\alpha$$

$\Delta\alpha_h$ Change on AoA due to bending

$\Delta\alpha$ Elastic Torsion

- Bending & Torsion DOF are coupled thru the aerodynamic forces
- Straight wings → $\Delta\alpha_h = 0$ → Bending & Torsion are uncoupled

DIVERGENCE SPEED
AEROELASTIC EQUATIONS

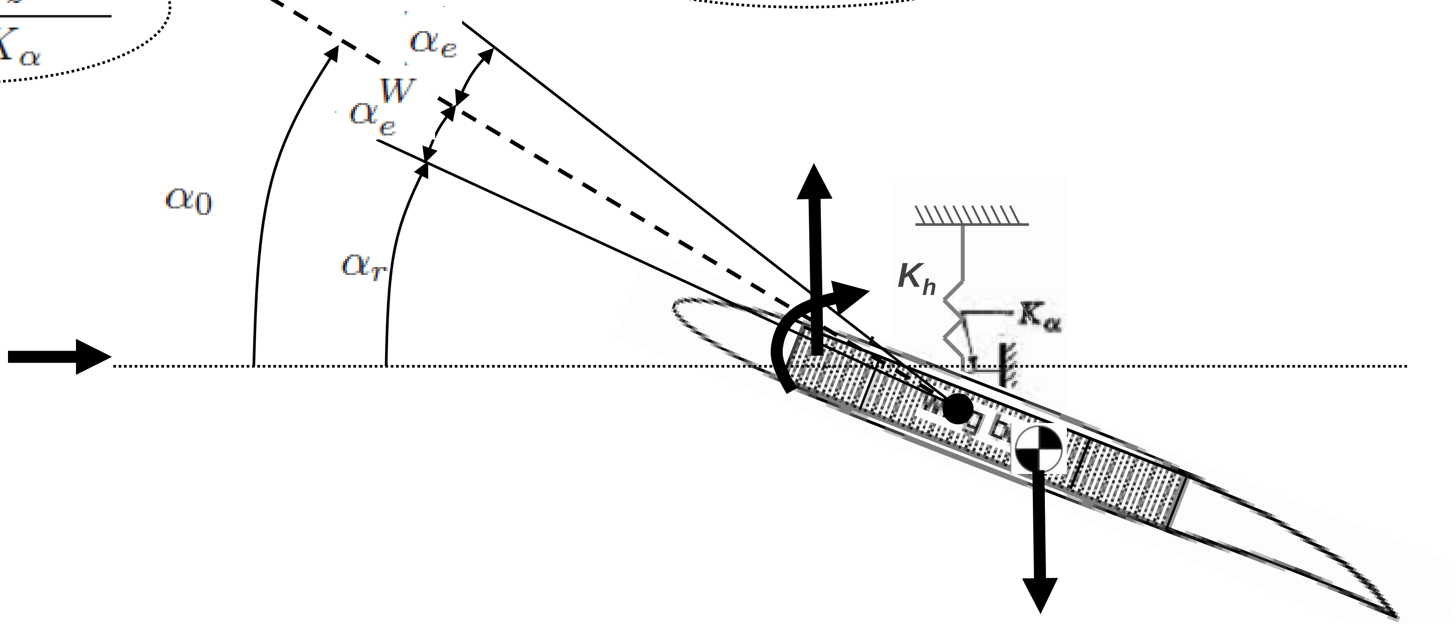
- For the sake of simplicity, let's assume "*straight wing*" and "*δ=0*"
- Linear aerodynamics: lift proportional to angle of attack

$$K_\alpha \Delta\alpha = M_{AC} + e \cdot L + d \cdot N_z \cdot W$$

$$K_\alpha \left(\alpha_e + \alpha_e^W\right) = q_\infty S c C_{MAC} + q_\infty S e C_{L\alpha} \left(\underbrace{\alpha_r + \alpha_e^W} + \alpha_e\right) + d \cdot N_z \cdot W$$

$$\alpha_e^W = \frac{d \cdot N_z \cdot W}{K_\alpha}$$

$$\alpha_0 = \alpha_r + \alpha_e^W$$



- The jig-shape: unstrained aircraft shape when supported in the jigs during manufacture without inertia or aerodynamic forces
- Physical meaning of different angles of attack

$$K_\alpha \Delta\alpha = M_{AC} + e \cdot L + d \cdot N_z \cdot W$$

$$K_\alpha \left(\alpha_e + \alpha_e^W\right) = q_\infty S c C_{MAC} + q_\infty S e C_{L\alpha} \left(\underbrace{\alpha_r + \alpha_e^W}_{} + \alpha_e\right) + d \cdot N_z \cdot W$$

$$\alpha_e^W = \frac{d \cdot N_z \cdot W}{K_\alpha}$$

$$\alpha_0 = \alpha_r + \alpha_e^W$$

$$K_\alpha \alpha_e = q_\infty S c C_{MAC} + q_\infty S e C_{L\alpha} \left(\alpha_0 + \alpha_e\right)$$

$$\left(K_\alpha - q_\infty S e C_{L\alpha}\right) \alpha_e = q_\infty S c C_{MAC} + q_\infty S e C_{L\alpha} \alpha_0$$

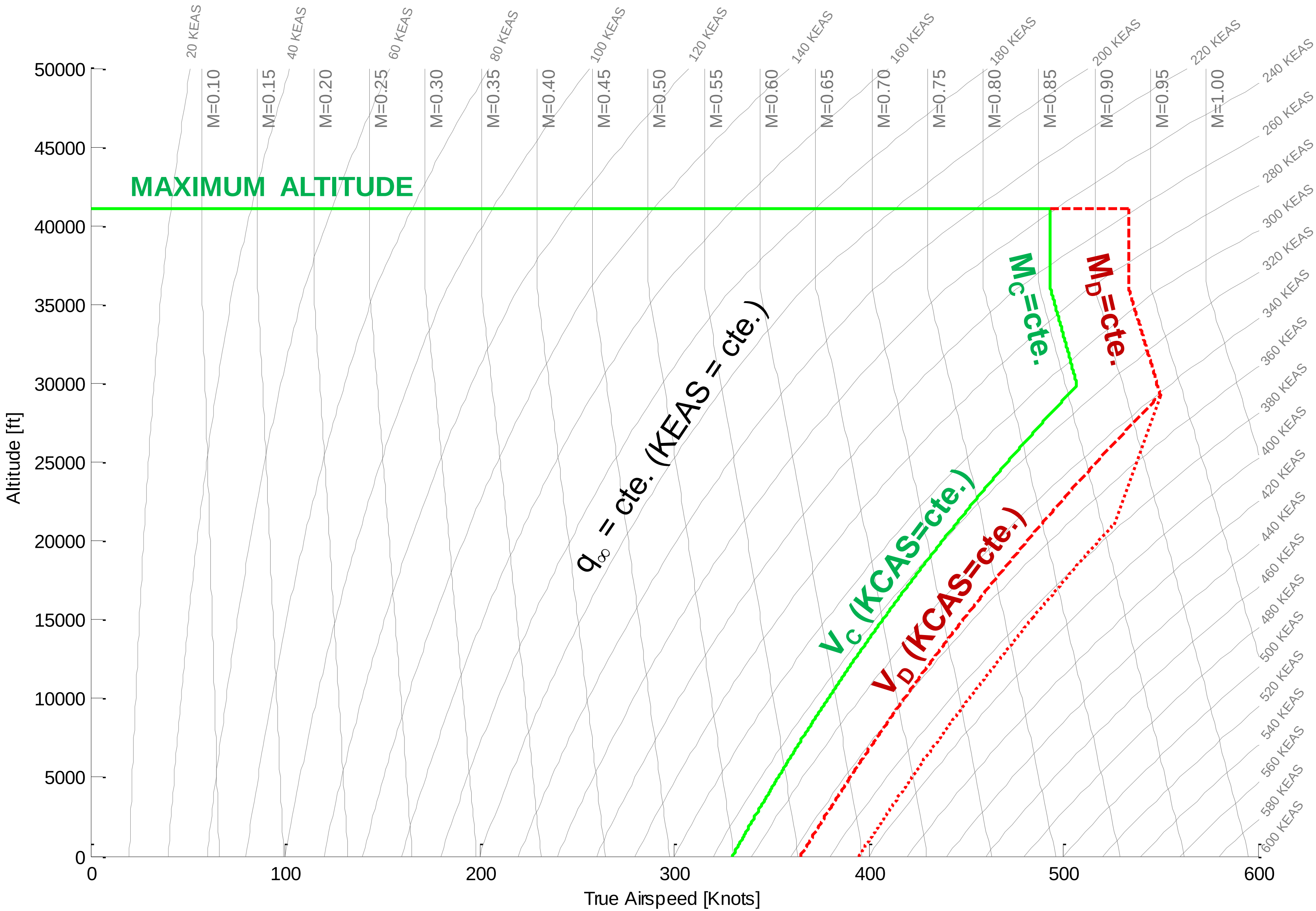
$$\alpha_e = \frac{q_\infty S c C_{MAC} + q_\infty S e C_{L\alpha} \alpha_0}{K_\alpha - q_\infty S e C_{L\alpha}} = q_\infty \frac{S e C_{L\alpha}}{K_\alpha} \frac{\alpha_0 + \frac{c}{e} \frac{C_{MAC}}{C_{L\alpha}}}{1 - q_\infty \frac{S e C_{L\alpha}}{K_\alpha}}$$

$$\alpha_e = \frac{\hat{q}}{1 - \hat{q}} \left(\alpha_0 + \frac{c}{e} \frac{C_{MAC}}{C_{L\alpha}}\right)$$

$$q_D = \frac{K_\alpha}{S e} \frac{1}{C_{L\alpha}}$$

- Does divergence speed depend on the aircraft mass state ?
- Does aerodynamic effectiveness depend on the aircraft mass state ?
- Does aerodynamic effectiveness depend on the jig shape ? … depend on the A/C angle of attack ?
- $C_{L\alpha}$ depends on Mach number → implicit equation

DYNAMIC PRESSURE
ISO-q LINES



- ❑ Mitsubishi A6M Zero vs Grumman F4F Wildcat → Japan 01 – USA 00
  - ► F4F Wildcat had a worse ratio Power / Weight
  - ► Pilots of Zero knew it and performed a vertical climbing maneuver
- ❑ Grumman F6F Hellcat to defeat Zero
  - ► More power to maintain the vertical climbing maneuver as the Zero
  - ► Dive maneuver to defeat the Zero that had problems with aileron reversal

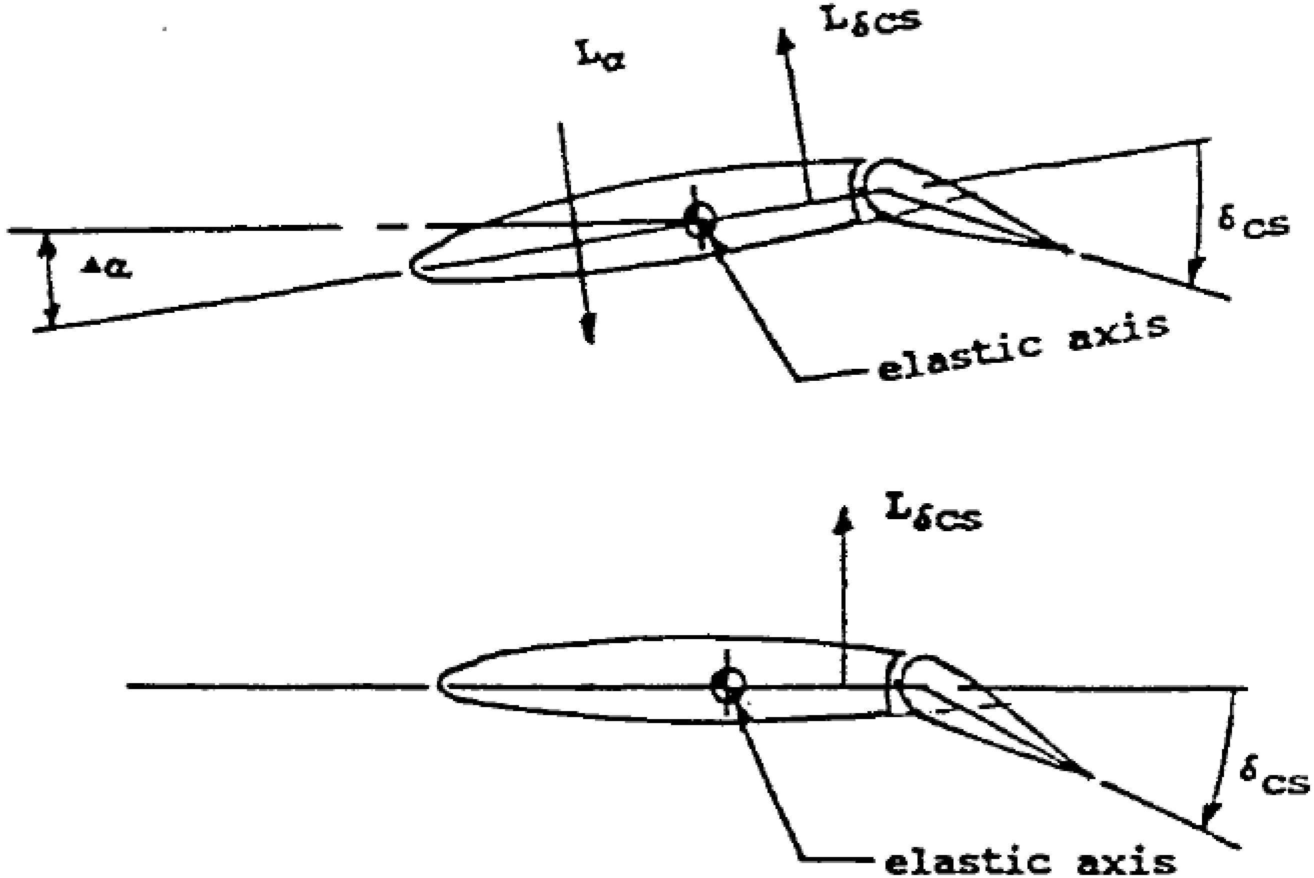


Fig. 13.4 Control surface reversal due to aeroelasticity. (Reversal will occur when the aerodynamic lift produced by the control surface is overcome by the aerodynamic loading due to aeroelasticity.) L_{α} = lift due to change in wing angle of attack; $L_{\delta CS}$ = lift due to control surface; δ_{CS} = control surface angle; $\Delta\alpha$ = change in angle of attack due to bending and torsion.

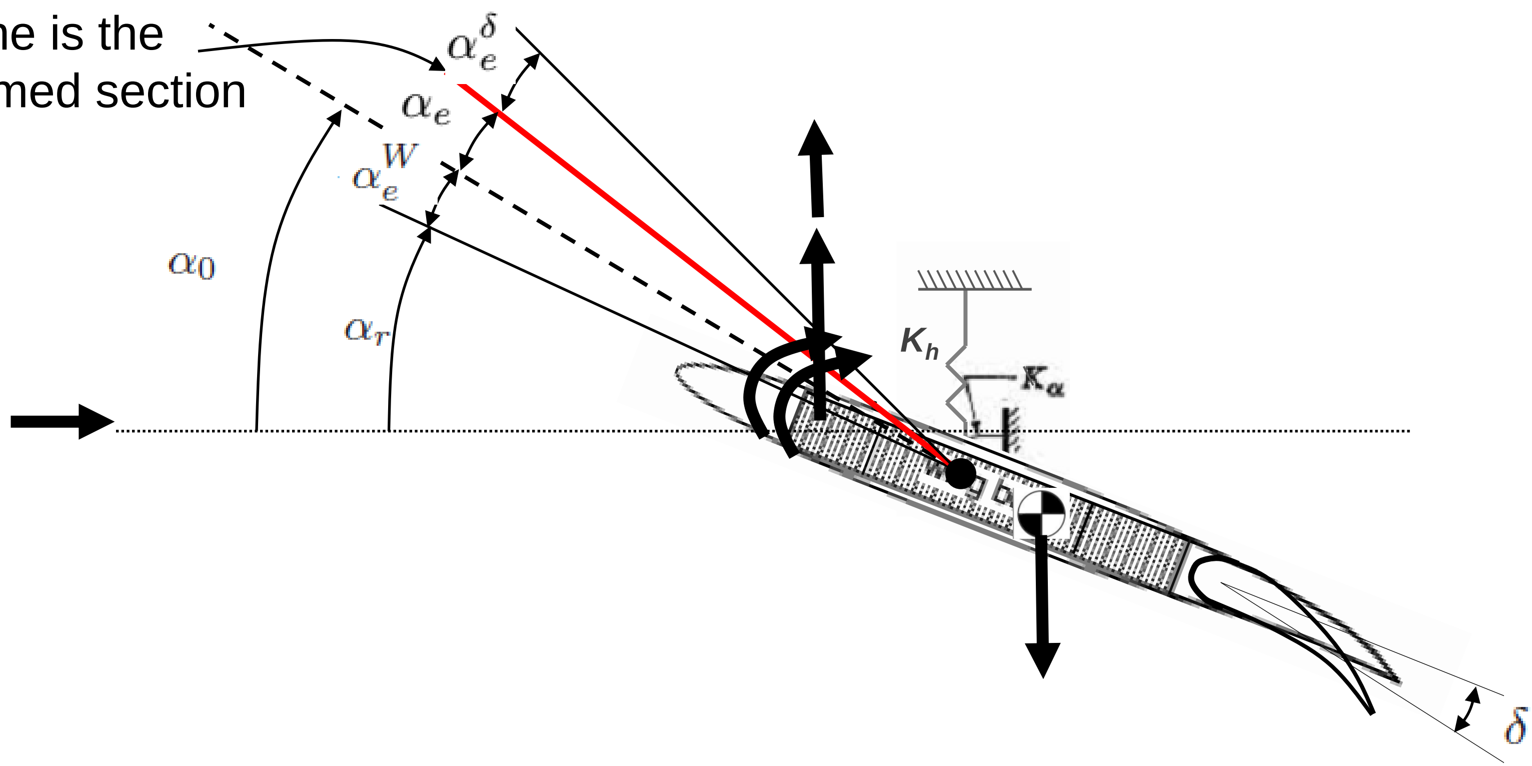
δ

- Effect of control surface deflection is calculated with respect to the deformed section due to angle of attack:

$$K_\alpha \alpha_e^\delta = q_\infty SeC_{L\delta}\delta + q_\infty ScC_{MAC\delta}\delta + q_\infty SeC_{L\alpha}\alpha_e^\delta$$

$$\left(K_\alpha - q_\infty SeC_{L\alpha}\right)\alpha_e^\delta = q_\infty Se\left(C_{L\delta} + \frac{c}{e}C_{MAC\delta}\right)\delta$$

Reference line is the elastic-deformed section



$$K_\alpha \alpha_e^\delta = q_\infty SeC_{L\delta}\delta + q_\infty ScC_{MAC\delta}\delta + q_\infty SeC_{L\alpha}\alpha_e^\delta$$

$$\left(K_\alpha - q_\infty SeC_{L\alpha}\right)\alpha_e^\delta = q_\infty Se\left(C_{L\delta} + \frac{c}{e}C_{MAC\delta}\right)\delta \Rightarrow \boxed{\alpha_e^\delta = \frac{q_\infty Se\left(C_{L\delta} + \frac{c}{e}C_{MAC\delta}\right)}{K_\alpha - qSeC_{L\alpha}}\delta}$$

$$\Delta L_{rig} = q_\infty SC_{L\delta}\delta$$

$$\Delta L_{flex} = q_\infty SC_{L\delta}\delta + q_\infty SC_{L\alpha}\alpha_e^\delta$$

CONTROL SURFACE EFFECTIVENESS

$$\frac{\Delta L_{flex}}{\Delta L_{rig}} = \frac{q_\infty SC_{L\delta}\delta + q_\infty SC_{L\alpha}\alpha_e^\delta}{q_\infty SC_{L\delta}\delta} = 1 + \frac{C_{L\alpha}}{C_{L\delta}}\frac{\alpha_e^\delta}{\delta} = 1 + \frac{C_{L\alpha}}{C_{L\delta}}q_\infty Se\frac{C_{L\delta} + \frac{c}{e}C_{MAC\delta}}{K_\alpha - qSeC_{L\alpha}} =$$

$$= \frac{K_\alpha - q_\infty SeC_{L\alpha} + q_\infty SeC_{L\alpha} + q_\infty Sc\frac{C_{L\alpha}C_{MAC\delta}}{C_{L\delta}}}{K_\alpha - qSeC_{L\alpha}} = \frac{K_\alpha + q_\infty Sc\frac{C_{L\alpha}C_{MAC\delta}}{C_{L\delta}}}{K_\alpha - q_\infty SeC_{L\alpha}} =$$

$$= \frac{1 + q_\infty \frac{Sc}{K_\alpha}\frac{C_{L\alpha}C_{MAC\delta}}{C_{L\delta}}}{1 - q_\infty \frac{Se}{K_\alpha}C_{L\alpha}} \Rightarrow q_R = -\frac{K_\alpha}{Sc}\frac{C_{L\delta}}{C_{L\alpha}C_{MAC\delta}}$$

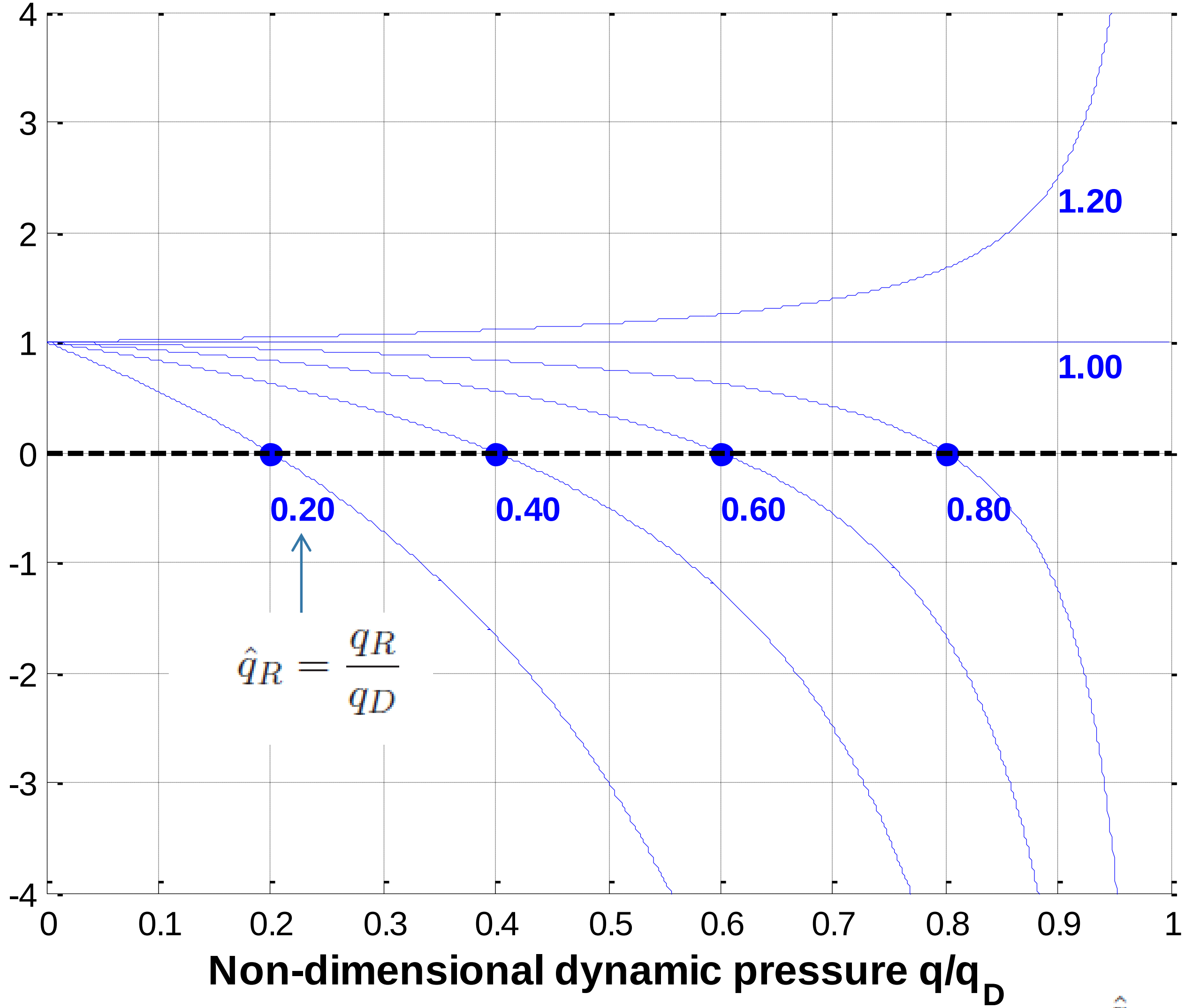
$$\frac{\Delta L_{flex}}{\Delta L_{rig}} = \frac{1 + q_\infty \frac{Sc}{K_\alpha}\frac{C_{L\alpha}C_{MAC\delta}}{C_{L\delta}}}{1 - q_\infty \frac{Se}{K_\alpha}C_{L\alpha}} = \frac{1 - \frac{q_\infty}{q_R}}{1 - \frac{q_\infty}{q_D}} \Rightarrow \boxed{\frac{\Delta L_{flex}}{\Delta L_{rig}} = \frac{1 - \frac{\hat{q}}{\hat{q}_R}}{1 - \hat{q}}}$$

$$\hat{q} = \frac{q_\infty}{q_D}$$

$$\hat{q}_R = \frac{q_R}{q_D}$$

CONTROL SURFACE EFFECTIVENESS AS FUNCTION OF RATIO q_R / q_D

$$\frac{\Delta L_{flex}}{\Delta L_{rig}} = \frac{1 - \frac{\hat{q}}{\hat{q}_R}}{1 - \hat{q}}$$

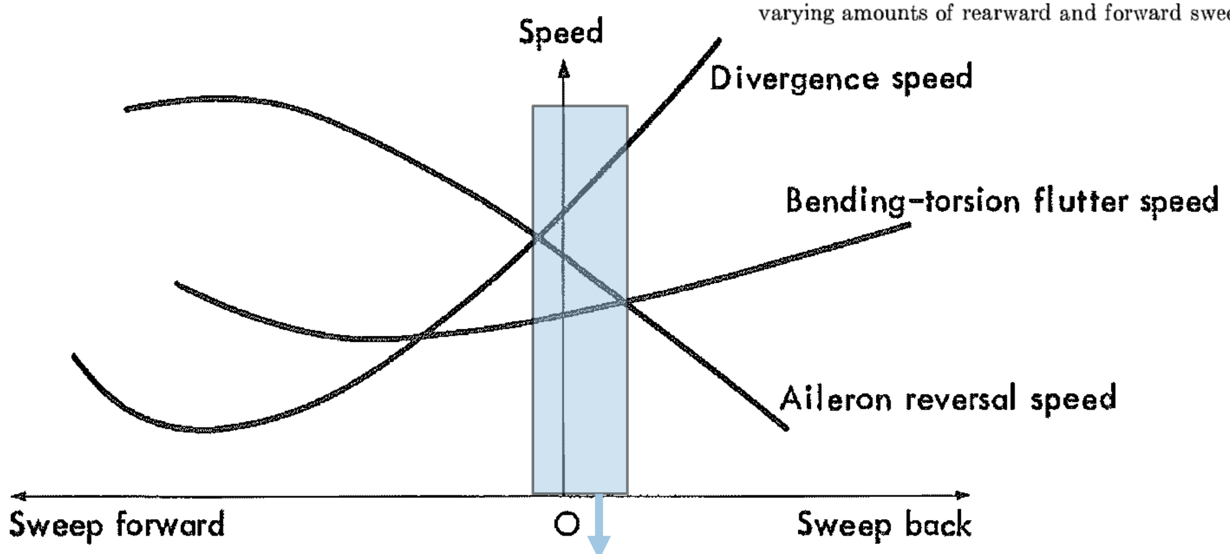


$$\hat{q} = \frac{q}{q_D}$$

- $q_R/q_D=1$: Effectiveness is maintained till catastrophic failure at $q=q_D$
- $q_R/q_D < 1$ vs. $q_R/q_D>1$: Which value is the best for an safe design ?

COMPARISON OF WING CRITICAL SPEEDS
(extracted from Bisplinghoff, “Aeroelasticity”)

1–4 Comparison of wing critical speeds. We have seen in the previous sections that a conventional wing has three critical speeds, each of which is important to the designer. They are the flutter speed, the divergence speed, and the aileron reversal speed. A comparison of their relative values is a necessary process in the design of a wing. In the case of straight unswept wings of conventional construction, wing torsional divergence usually occurs at a speed higher than aileron reversal speed, which is in turn higher than the bending-torsion flutter speed. In the case of swept-forward wings, the divergence speed can be expected to be lower than the flutter speed, which is in turn lower than the aileron reversal speed. For swept-back wings, the aileron reversal speed is lower than the flutter speed, which is in turn lower than the divergence speed. Figure 1–8 shows qualitatively the relation between critical speeds for a typical wing with varying amounts of rearward and forward sweep.



Grumman X-29

Fairchild-Republic A-10 Thunderbolt II

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